

Inference for Projections of Identified Sets

Hiroaki Kaido (Boston University)

Francesca Molinari (Cornell University)

Jörg Stoye (Cornell University)

We provide confidence regions for projections of identified sets that are not projections of confidence regions. The motivation for considering projections is that one is frequently interested in (i) constructing separate confidence regions for components of a partially identified vector, akin to the confidence regions standardly reported in multiple linear regression, or (ii) in testing simple linear hypotheses with regard to components of a partially identified vector. The motivation for not projecting confidence regions for the identified set – of which there are many that work under rather general conditions – is that such projection is conservative. The conservatism will be severe in high-dimensional problems.

In a framework of extremum estimation with set-valued argmin, our proposal is to report the projection of a k -level set of the sample criterion function, where k is chosen to achieve the desired coverage. In the salient special case of moment inequalities, this boils down to relaxing all moment inequalities by some amount c and reporting the projection of the set defined by the relaxed inequalities. The correct choice of k or c depends on the local geometry of the identified set around its support sets in the relevant directions, i.e. around the (not necessarily unique) elements of the set that correspond to extrema of the projection. Thus, the value will be different for one and the same d.g.p but different directions of projection.

A difficulty is that certain aspects of this local geometry cannot be pre-estimated with sufficient accuracy for the pre-estimation step to be ignorable. To achieve coverage that is uniform over certain interesting classes of d.g.p.'s, as well as directions of projection, we therefore resort to locally conservative distortion of this geometry. One of the distortions we use is very similar to Generalized Moment Selection (Andrews and Soares, *Econometrica* 2010), but others (e.g. for the case of distinct local maxima that might be global maxima) are novel to our approach.

We currently (at time of submission of this abstract) achieve pointwise validity of confidence intervals in the most general setting, uniform validity under reasonable conditions in the moment inequalities setting, and uniform validity of a substantially simplified procedure in the special case where all moment inequalities are linear. Results are illustrated in the context of a 2x2 entry game à la Tamer (*Review of Economic Studies* 2003), using level-2 rationalizability as solution concept. Our approach compares favorably to some other methods found in the literature.